Complements

If A and B are sets, the <u>difference</u> between A and B

is $A - B = \{x \in A \mid x \notin B\}$ (not necessary to assume $B \subseteq A$)

Ex: $\{x \in \mathbb{N} \mid x \text{ is prime}\} - \{y \in \mathbb{N} \mid y \text{ is odd}\} = \{2\}$

For now,
Unless otherwise specified, assume sets mentioned are all subsets of the same set E.

Then the <u>complement</u> of A is A' (or $\bar{A}$, or A^c) and it is defined to be $E - A$.

(usually when dealing w/ specific sets, it will be clear what "E" is)

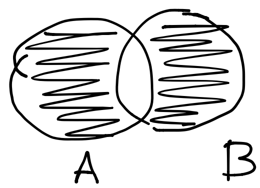
Ex: In $\mathbb{Z}$, the complement of the set of odd #s is the set of even #s.

<u>Basic Facts about complements</u>

- $(A')' = A$
- $\emptyset' = E$
- $A \cap A' = \emptyset$, $A \cup A' = E$
- $A \subset B \iff B' \subset A'$
- $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$ (DeMorgan's Laws)

The <u>symmetric difference</u> of A and B is $A + B$, defined

$$A + B = (A - B) \cup (B - A)$$



The Power set of a set

If A is a set, the <u>power set</u> of A, denoted $\mathcal{P}(A)$, is the set of all subsets of A. i.e.

$$\mathcal{P}(A) = \{B \mid B \subseteq A\}$$

Ex:

- $\mathcal{P}(\{x,y\}) = \{\emptyset, \{x\}, \{y\}, \{x,y\}\}$ (4 elements)
- $\mathcal{P}(\emptyset) = \{\emptyset\}$ (1 element)
- $\mathcal{P}(\{a,b,c\}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$ 8 elements
- For any set A, $A, \emptyset \in \mathcal{P}(A)$

Claim: If X is a finite set w/ n elements, then $\mathcal{P}(X)$ has 2^n elements.

(if $X = \{a_1, a_2, \ldots, a_n\}$, then for each subset, a_i is either in the subset or not — 2 choices for each i)

Power set of infinite sets? We will see that the power set of an infinite set is "bigger" than the original set. e.g. $\mathcal{P}(\mathbb{Z})$ has the same amount of elements as $\mathbb{R}$.

Claim: If E and F are sets, then

1. $\mathcal{P}(E) \cap \mathcal{P}(F) = \mathcal{P}(E \cap F)$

2. $\mathcal{P}(E) \cup \mathcal{P}(F) \subset \mathcal{P}(E \cup F)$

Pf: 1. Let $A \in \mathcal{P}(E) \cap \mathcal{P}(F)$. Then $A \subset E$ and $A \subset F$, so $A \subset (E \cap F)$. Thus $A \in \mathcal{P}(E \cap F)$.

Now suppose $B \in \mathcal{P}(E \cap F)$. Then $B \subseteq (E \cap F)$, so $B \subseteq E$ and $B \subseteq F \Rightarrow B \in \mathcal{P}(E) \cap \mathcal{P}(F)$.

2. Generalization in HW.

More facts:

- $\bigcap_{X \in \mathcal{P}(E)} X = \phi$
- $E \subset F \Rightarrow \mathcal{P}(E) \subset \mathcal{P}(F)$
- $\bigcup_{X \in \mathcal{P}(E)} X = E$.

Cartesian Products

If A and B are sets, their <u>Cartesian product</u>, $A \times B$, is defined

$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

Ex:

- $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2 =$ "the xy-plane"
- $\{x,y\} \times \{x,z\} = \{(x,x), (x,z), (y,x), (y,z)\}$

- $A \times \emptyset = \emptyset$